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RELATIVISTIC ELECTRON BEAM WITH A VARIABLE DEGREE OF CHARGE NEUTRALIZATION

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A rather large number of theoretical papers have appeared on the steady states of relativistic beams which are uniform along the axis (cf. reviews [1, 2]), but under actual experimental conditions the degree of charge neutralization of a beam may vary considerably along the tube axis. In quasistationary and stationary systems the ion density may vary appreciably along the length when there are longitudinal sinks [3]. In addition, a number of papers (e.g., [4]) consider the focusing of beams in a guide tube by a pressure gradient along the beam. Thus, it is interesting to consider the equilibrium state of a beam for a variable degree of charge neutralization along the tube through which a quasisteady beam is propagated.

We assume that a beam with a current I is injected along the axis of a metal tube of length 2L with a density of neutral particles $n_0(z)$. The ion density $n_i(z)$ arising as a result of ionization will also vary. For simplicity we assume that the beam is narrow enough so that we can consider n_0, n_i, and n_e, independent of the radius ρ. We denote the characteristic scale of the nonuniformity of the n_i distribution by l, where $l \lesssim L$.

We assume that the density of neutral atoms, and consequently the ion density, is maximum at $z = 0$ ($-L \leq z \leq L$). By assumption the state of the beam varies from nearly forcefree at the ends of the tube toward quasineutral, but the conditions for quasineutrality and force-free motion are not satisfied exactly ($n_e > n_i \gtrsim n_e/\gamma^2$, where γ is the ratio of the total energy of an electron to its rest energy). Under these conditions it is required to find the characteristics of an axially symmetric beam ($n_e(z)$, $n_i(z)$, and $\gamma(z)$), taking account of the effect of self-fields on the motion of beam particles.

If **v** if the velocity of an electron, and **E** and **H** are the intensities of the electric and magnetic fields, the equation of motion of an electron can be written in the form

$$\dot{\mathbf{v}} = -\frac{e}{m\gamma}\left\{\mathbf{E} - \frac{1}{c^2}\mathbf{v}(\mathbf{vE}) + \frac{1}{c^2}[\mathbf{vH}]\right\}, \tag{1}$$

where e and m are the charge and mass of the electron, and c is the velocity of light.

We use the adiabatic approximation to solve (1); i.e., we assume that **E**, **H**, and γ vary slowly along the axis of the tube. Then we can set $E_z \approx 0$.

The projection of (1) along the direction of the radius ρ of a cylindrical coordinate system gives

$$\ddot{\rho} + \frac{e}{m\gamma}\left\{E_\rho\left(1 - \frac{\dot{\rho}^2}{c^2}\right) - \frac{\dot{z}}{c}H_\theta\right\} = 0. \tag{2}$$

The field components E_ρ and H_θ are $E_\rho = 2\pi e\rho(n_i - n_e)$ and $H_\theta = -2\pi e\rho n_e\beta_z$, where $n_e(z)$ is the electron density in the beam, $\beta_z = v_z/c$, and v_z is the average longitudinal velocity of the electrons. Then (2) can be written in the form

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$$\ddot{\rho} + \rho\Omega_0^2 - \rho\,\frac{\dot{\rho}^2}{c^2}\,\Omega_1^2 = 0, \tag{3}$$

$$\Omega_0^2 = \frac{2\pi e^2}{m\gamma}\left(n_i - \frac{n_e}{\gamma^2}\right), \qquad \Omega_1^2 = \frac{2\pi e^2 n_i}{m\gamma}.$$

If Ω_0 and Ω_1 are assumed constant Eq. (3) can be integrated to give

$$\frac{\Omega_1^2}{c^2}\left(\frac{d\rho}{dt}\right)^2 = \Omega_0^2 + \left(\frac{\Omega_1^2}{c^2}\,\dot{\rho}_0^2 - \Omega_0^2\right) e^{\frac{\Omega_1^2}{c^2}(\rho^2-\rho_0^2)} \tag{4}$$

where ρ_0 and $\dot{\rho}_0$ are respectively the initial radius and the initial transverse velocity of an electron.

The role of the nonlinear term in (3) can be estimated from (4). If the exponent is small its expansion and the subsequent averaging of the solution over the oscillations for large Ω_0 gives

$$\overline{\rho^2} = \frac{1}{2}\left(\rho_0^2 + \frac{\dot{\rho}_0^2}{\Omega_0^2 - \frac{\Omega_1^2}{c^2}\dot{\rho}_0^2}\right), \qquad \overline{\dot{\rho}^2} = \frac{1}{2}\left\{\rho_0^2\left(\Omega_0^2 - \frac{\Omega_1^2}{c^2}\dot{\rho}_0^2\right) + \dot{\rho}_0^2\right\}.$$

It is clear from these relations that if the radius of the initial distribution is small ($\rho_0^2\Omega_1^2/c^2 \ll 1$), and the initial velocities of the particles are parallel to the axis ($\dot{\rho} = 0$), the radius always remains small and varies slowly only with a change in Ω_0, and the transverse velocities also remain small. The condition $\dot{\rho} \ll c$ implies $\rho_0 \ll c/\Omega_0$. To satisfy this inequality the concentration of electrons must not be too large, i.e., the beam currents must not be too large. Using the maximum value of Ω_0 at $z=0$ we have as an estimate

$$\rho_0^2 \ll \gamma mc^2/2\pi e^2 n_e,$$

i.e., a sufficient condition is

$$i \ll \gamma/2, \tag{5}$$

where $i = eI/mc^3$ and $I = \pi e\rho_0^2 n_e c$ is the beam current. We note also that in the range of parameters of interest to us we can set $\Omega_0 \approx \Omega_1$.

Thus, we assume that (5) is satisfied; i.e., the nonlinear term in (3) gives only a small correction to the results obtained.

If the condition $\gamma \gg 1$ is satisfied, $\beta_z \approx 1$ for all the beam particles. Then in (3) the variable t can be changed to z:

$$\rho'' + \Omega^2(z)\rho = 0, \tag{6}$$

where Ω in general depends on $\dot{\rho}$ also:

$$\Omega^2 = \frac{2\pi e^2}{\gamma mc^2}\left(n_i\left(1 - \frac{\dot{\rho}^2}{c^2}\right) - \frac{n_e}{\gamma^2}\right). \tag{7}$$

We consider the case when (6) can be solved in the WKB approximation, i.e., when $\Omega^2 \gg \Omega^1$. Since Ω can be estimated as $\Omega \approx \omega_e/c$, where $\omega_e^2 = 2\pi e^2 n_e/m\gamma$ and $\Omega' \approx \Omega/l$, a sufficient condition for the applicability of the WKB approximation is $l \gg c/\omega_e$, which is satisfied for a very broad range of parameters.

The solution of (6) has the form

$$\rho = \frac{A_1}{\sqrt{\Omega}}\cos\theta(z) + \frac{A_2}{\sqrt{\Omega}}\sin\theta(z), \qquad \frac{d\theta}{dz} = \Omega(z). \tag{8}$$

Because of the large values of θ only quantities averaged over the oscillations have a physical meaning. From (8) the averages of the squares are

$$\overline{\rho^2}(z) = C_0/\Omega(z), \; \overline{\dot{\rho}^2}(z) = C_1\Omega(z). \tag{9}$$

We note that the following inequalities must be satisfied:

$$C_0 > 0, \; C_1 > 0, \; \Omega(z) > 0.$$

Equation (7) for $\Omega(z)$, which contains a nonlinear term, can also be averaged over the oscillations to give

$$\Omega^2(z) = \frac{2\pi e^2}{\gamma mc^2}\left(n_i(1 - C_1\Omega(z)) - \frac{n_e}{\gamma^2}\right). \tag{10}$$

Equation (10) takes account of the nonlinear character of the transverse oscillations of an electron, but is valid only when the correction related to the constant C_1 is small, which is necessary to satisfy the inequality $\overline{\Delta\Omega^2} \ll \overline{\Omega}^2$.

We note also that in (10) Ω depends on the initial conditions, which is generally the case for nonlinear oscillations.

Equation (10) relates the functions $\Omega(z)$, $\gamma(z)$, $n_i(z)$, and $n_e(z)$. The value of $\gamma(z)$ can be found from $\gamma(z) = \gamma_R + e\Phi/mc^2$, where γ_R is determined in terms of the potential of the tube wall.

It has been assumed that the degree of charge neutralization of the beam increases from the ends of the tube toward the middle but does not reach unity, since in the case under consideration the charge of the beam must be large enough so that secondary electrons produced in ionization reach the wall of the tube.

For a potential $\Phi(z)$ and a very thin beam ($\ln(R/a) \gg 1/2$), we have the relation

$$\Phi(z) = -2\pi e a^2(z)(n_e(z) - n_i(z))\ln(R/a(z)),$$

so that

$$\gamma(z) = \gamma_R - \frac{2\pi e^2 a^2(z)}{mc^2}(n_e(z) - n_i(z)) \ln \frac{R}{a(z)}. \tag{11}$$

From Eq. (9) the radius of the beam is given by

$$a^2(z) = C_3/\Omega(z).$$

For a closed system the beam current is constant

$$I = \pi a^2(z) e n_e(z) c = \text{const},$$

from which

$$n_e = C_4/a^2(z), \tag{12}$$

and

$$n_e = C_5\Omega(z). \tag{13}$$

Eliminating $\Omega(z)$ from (13) and (10)

$$n_e(z) = \sqrt{\frac{(C_5C_6)^2}{4\gamma^6} + \frac{C_6 n_i}{\gamma}} - \frac{C_5C_6}{2\gamma^3}, \tag{14}$$

where $C_1, \ldots, C_5$ are positive constants which depend on the initial conditions, and $C_6 = 2\pi e^2/mc^2$.

Thus, Eqs. (11), (12), and (13) suffice for the determination of the electron density in the beam and the potential distribution for a given ion density $n_i(z)$. Because of the presence of the factor $\ln(R/a)$, however, these relations cannot be expressed explicitly.

One general conclusion can be drawn concerning the character of the functions $n_e(z)$ and $\gamma(z)$, namely, that if n_i is symmetric about $z = 0$, n_e and γ will also be symmetric.

In principle the relations obtained above express γ and n_e in terms of the ion density, which can be determined if the densities n_0 and n_e and the potential distribution are known.

For simplicity we assume that z varies from 0 to L. Then for $n_i(z)$ we have

$$n_i(z) = \int_0^z \frac{\nu_i(\xi)\, n_e(\xi)\, d\xi}{c\sqrt{2\frac{m}{M}(\gamma(\xi) - \gamma(z))}}, \tag{15}$$

where M is the mass of an ion and $\nu_i(z)$ is the frequency of collisions of beam electrons with neutral atoms leading to the formation of ions:

$$\nu_i(z) = n_0(z)\sigma_i c,$$

where σ_i is the ionization cross section.

Equation (15) is a consequence of the equation of continuity for ions and is valid only for a monotonic function $\gamma(z)$ in the integrand.

Henceforth we shall be interested mainly in a range of parameters corresponding to characteristics of the beam close to $z = 0$. We omit the term $n_e \gamma^2$ on the right-hand side of (10), so that instead of (14) we obtain the simpler expression

$$n_e(z) \simeq n_e(0)\left(\frac{n_i(z)\gamma(0)}{n_i(0)\gamma(z)}\right)^{1/2}. \tag{16}$$

Equation (1) can be written in the form

$$\gamma(z) = \gamma_R - i\Lambda(z)(1 - n_i(z)/n_e(z)), \tag{17}$$

$\Lambda(z) = 2\ln(R/a(z))$, where because of the very slowly varying logarithmic function $\Lambda \approx \text{const} = \Lambda_0$, Equations (15)-(17) form a self-consistent system of equations which uniquely determines the required quantities in terms of the density of neutral atoms $n_0(z)$.

It is convenient to introduce a new function

$$n_i(z)/n_e(z) = 1 - \alpha(z), \quad n_i(0)/n_e(0) = 1 - \alpha_0,$$
$$\alpha(z) = \alpha_0 + \alpha_1(z).$$

All the required characteristics n_i, n_e, and γ can be expressed in terms of $\alpha(z)$:

$$n_i(z) = \frac{(1-\alpha)^2}{1 - \frac{i\Lambda\alpha_1(z)}{\gamma_0}} \frac{n_e(0)}{(1-\alpha_0)}; \tag{18}$$

$$n_e(z) = \frac{1-\alpha}{1 - \frac{i\Lambda\alpha_1(z)}{\gamma_0}} \frac{n_e(0)}{(1-\alpha_0)}; \tag{19}$$

$$\gamma(z) = \gamma_0 - i\Lambda\alpha_1(z). \tag{20}$$

The function $\alpha(z)$ is determined from the nonlinear integral equation (15). In general this equation is very difficult to solve, but if we use the condition $\alpha_1(z) \ll 1 - \alpha_0$, which is valid for small z, and assume $i\Lambda/\gamma_0 \ll 1$, the equation takes the simpler form

$$1 = \frac{\sigma_0}{\sqrt{i\Lambda}} \int_0^z \frac{n_0(\xi)\, d\xi}{\sqrt{\alpha_1(z) - \alpha_1(\xi)}} \frac{1}{1-\alpha_0}, \quad \sigma_0 = \sigma_i \sqrt{\frac{M}{2m}}.$$

By using the expansion $\alpha_1(\xi) = \alpha_1(z) + (\xi - z)\alpha_1(z)$:

$$\alpha_1(z) = \frac{\sigma_0^2}{i\Lambda} \int_0^z K_1^2(\xi)\, d\xi \frac{1}{(1-\alpha_0)^2},$$

where

$$K_1(z) = \int_0^z \frac{n_0(\xi)\, d\xi}{\sqrt{z-\xi}}.$$

On the basis of Eqs. (18)-(20) a qualitative analysis can now be given of the behavior of $n_i(z)$, $n_e(z)$ and $\gamma(z)$ near $z = 0$. All three functions have maxima at $z = 0$. The total density falls off twice as rapidly as the electron density, and because of the condition $i\Lambda/\gamma_0 \ll 1$ the relative falloff of $\gamma(z)$ is very slow.

A more exact analysis requires a numerical solution of the equations presented.

In summary we have obtained a self-consistent system of equations for determining the state of nonuniformity along the axis of an axially symmetric relativistic electron beam, taking account of the ionization of the residual gas.

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